AI Enabled Control Engineering

Laboratory Report 2: LQR Control of a Rotary Inverted Pendulum

Group:	Group XX
Students:	Name 1; Name 2; Name 3; Name 4
Student IDs:	ID 1; ID 2; ID 3; ID 4
Submission date:	YYYY-MM-DD

Submit one group ZIP through one designated representative. Include the report PDF, all original CSV logs and one original PNG result figure for every CSV log.

Teaching Assistants: Zhixiang Ju and Haozhe Wang

Submission Requirements – Remove Before Final Submission

Before submission, change `\instructionstrue` to `\instructionsfalse`. Remove all placeholder text and retain only the group's own calculations, results and analysis.

Group submission and contribution-based individual grading
Each group submits one report ZIP through one designated representative. Groups may contain two, three or four members. Group size does not affect the grading criteria or the total group score. However, the individual scores of group members may be adjusted according to the agreed contribution percentages, so that relative effort and participation are reflected fairly.
It is strongly discouraged to assign the entire experiment and report to one member, except when another member could not attend the relevant laboratory session for a valid reason. State each member's work and percentage clearly. The percentages must sum to 100%. The teaching assistants will not determine or arbitrate the group's internal division of work; disagreements must be resolved within the group before submission.

Required ZIP structure

```
GroupXX_LabReport2.zip
|-- GroupXX_LabReport2.pdf
|-- data/
|   |-- P2_SIM_CONT_R1.csv
|   |-- P2_SIM_DISC_R1.csv
|   |-- P3_HW_DISC_R1.csv
|   |-- P5_SIM_PID_R1.csv
|   `-- P5_HW_PID_R1.csv
`-- figures/
    `-- one original PNG for every CSV, with matching identifiers
```

Every file referenced in the report must be included and open correctly.

Controlled comparisons
When comparing continuous-design and discrete-design LQR, keep the nonlinear plant, initial condition, PWM limit, swing-up parameters, blend, duration and noise settings unchanged. When comparing simulation and hardware, use the same discrete gain and matching controller settings. When comparing LQR with Class-8 dual PID, use the group's own previously tuned PID gains rather than another group's values.

Scientific reporting
Show calculation steps and units. Every figure must have a caption and be discussed in the text. Report controller settings with sufficient precision to reproduce the experiment. Do not show only a favourable run; include all runs used to calculate summary statistics and discuss unsuccessful captures or outliers honestly.

Contents

Group Member Contributions

The percentages must sum to 100%. Group size does not change the grading criteria or total group score; the declared contribution percentages may affect the relative individual scores within the group.

Table 1: Contribution of each group member

Member	**Name**	**Main contribution**	**Contribution (%)**
Member 1			
Member 2			
Member 3			
Member 4			
		Total	**100%**

Remove unused rows for groups with fewer than four members.

1 Part I: Continuous and Discrete LQR Gain Calculation

1.1 Model, state convention and weights

State the state order and reproduce the matrices used in the calculation:

$$\boldsymbol{x} = \begin{bmatrix}\theta & \dot{\theta} & \alpha & \dot{\alpha}\end{bmatrix}^T, \qquad \dot{\boldsymbol{x}} = A\boldsymbol{x} + Bu.$$

Record the selected matrices and sample time:

$$Q = \text{diag}(\_\_\_\_\_\_, \_\_\_\_\_\_, \_\_\_\_\_\_, \_\_\_\_\_\_), \qquad R = \_\_\_\_\_\_\_\_, \qquad T_s = \_\_\_\_\_\_\_\_.$$

1.2 Continuous-time CARE calculation

Show the performance index, CARE, calculation method and final P_c and K_c. Use the standard convention $u = -K_c x$.

Table 2: Continuous-design LQR result

Quantity	Numerical result
$K_{c,\theta}$	
$K_{c,\dot{\theta}}$	
$K_{c,\alpha}$	
$K_{c,\dot{\alpha}}$	
Closed-loop eigenvalues of $A - BK_c$	

1.3 ZOH discretisation and DARE calculation

First convert the continuous-time model into a discrete-time model using zero-order-hold (ZOH) discretisation with

$$T_s = 0.005 \text{ s}.$$

For the input u_k being constant within each sampling interval,

$$x_{k+1} = A_d x_k + B_d u_k,$$

where

$$A_d = e^{AT_s},$$

and

$$B_d = \int_0^{T_s} e^{A\tau} B \, d\tau.$$

Students should show how A_d and B_d are obtained numerically.
For the discrete-time LQR design, consider the cost function

$$J_d = \sum_{k=0}^{\infty} (x_k^T Q x_k + u_k^T R u_k).$$

The optimal control law has the form

$$u_k = -K_d x_k.$$

Starting from the Bellman optimality equation, derive the discrete Riccati equation:

$$P_d = A_d^T P_d A_d - A_d^T P_d B_d (R + B_d^T P_d B_d)^{-1} B_d^T P_d A_d + Q.$$

After obtaining the stabilising solution P_d, derive the feedback gain:

$$\boxed{K_d = (R + B_d^T P_d B_d)^{-1} B_d^T P_d A_d}.$$

Students should report the calculation method (Python/MATLAB or equivalent), the obtained P_d, and the final gain vector using the convention

$$u_k = -K_d x_k.$$

Table 3: Discrete-design LQR result

Quantity	Numerical result
P_d	
$K_{d,\theta}$	
$K_{d,\dot{\theta}}$	
$K_{d,\alpha}$	
$K_{d,\dot{\alpha}}$	
Closed-loop eigenvalues of $A_d - B_d K_d$	

1.4 Why the two gains differ

Explain why solving the CARE before sampling and solving the DARE after ZOH discretisation do not generally produce identical gains.

2 Part II: Nonlinear Simulation Results

2.1 Common settings and experimental fairness

List the common initial condition, duration, PWM limit, swing PWM, enter/exit angles, soft-blend parameter and the required Class-8 noise settings. Confirm that only the LQR gain vector was changed between the two required simulations.

Table 4: Common nonlinear-simulation settings

Setting	Value	Setting	Value
Initial condition		Duration	
PWM limit		Swing PWM	
Enter angle		Exit angle	
Soft blend λ		Control interval	
θ noise (μ, σ)		α noise (μ, σ)	
PWM noise (μ, σ)		Number of runs	

2.2 Simulation protocol and stable-stage definition

Evaluate the continuous-design gain K_c and the discrete-design gain K_d using the same nonlinear simulation environment.

For each controller, complete ten independent simulation runs using one fixed set of controller and experiment parameters. Record failed runs as extttFail and use "–" for unavailable stable-stage quantities; do not replace them with additional favourable runs. The following settings must remain identical for the two controllers:

- initial condition and simulation duration;

- sampling period;
- swing-up PWM and PWM limit;
- LQR entry and exit angles;
- soft-blend coefficient;
- Gaussian observation-noise and PWM-noise parameters.

Only the LQR gain vector may be changed between the two sets of experiments.

For this report, the final stable stage is defined as the final continuous interval during which the pendulum remains inside the LQR stabilization region and the controller remains in the pure LQR mode until the end of the run. The swing-up stage and the soft-blend transition stage must not be included in the stable-stage statistics.

2.3 Continuous-design LQR simulation

Use the continuous-design gain K_c to complete ten independent nonlinear simulation runs with fixed parameters.

Save every simulation run as one matched CSV/PNG pair. A recommended naming convention is

```
lqr_sim_continuous_01.csv
lqr_sim_continuous_01.png
```

and similarly for runs 02–10.

Table 5: Ten repeated simulation experiments using the continuous-design gain K_c.

Trial	Stable time/s	Mean $\lvert\alpha\rvert$/rad	Std $\lvert\alpha\rvert$/rad	Mean $\lvert PWM\rvert$	Std $\lvert PWM\rvert$	Max overshoot/rad
1	–	–	–	–	–	–
2	–	–	–	–	–	–
3	–	–	–	–	–	–
4	–	–	–	–	–	–
5	–	–	–	–	–	–
6	–	–	–	–	–	–
7	–	–	–	–	–	–
8	–	–	–	–	–	–
9	–	–	–	–	–	–
10	–	–	–	–	–	–
Across-run mean	–	–	–	–	–	–
Across-run std.	–	–	–	–	–	–

2.4 Discrete-design LQR simulation

Replace K_c with the discrete-design gain K_d and repeat the same ten-run nonlinear simulation experiment.

All simulation, swing-up and noise settings must remain unchanged.

Save every run as one matched CSV/PNG pair using a naming convention such as

```
lqr_sim_discrete_01.csv
lqr_sim_discrete_01.png
```

and similarly for runs 02–10.

Table 6: Ten repeated simulation experiments using the discrete-design gain K_d.

Trial	Stable time/s	Mean $\lvert\alpha\rvert$/rad	Std $\lvert\alpha\rvert$/rad	Mean $\lvert PWM\rvert$	Std $\lvert PWM\rvert$	Max overshoot/rad
1	–	–	–	–	–	–
2	–	–	–	–	–	–
3	–	–	–	–	–	–
4	–	–	–	–	–	–
5	–	–	–	–	–	–
6	–	–	–	–	–	–
7	–	–	–	–	–	–
8	–	–	–	–	–	–
9	–	–	–	–	–	–
10	–	–	–	–	–	–
Across-run mean	–	–	–	–	–	–
Across-run std.	–	–	–	–	–	–

2.5 Definition of evaluation metrics

Use the same definitions as those adopted in Report 1:

- **Stable time**: the time when the pendulum enters the final pure-LQR stabilization stage and remains stable until the end of the run. Samples from the swing-up and soft-blend stages must not be included.
- **Mean** $|\alpha|$: the average absolute pendulum-angle error during the final stable stage,

$$\text{Mean}\,|\alpha| = \frac{1}{N_s}\sum_{k\in\mathcal{S}}|\alpha_k|,$$

  where $\mathcal{S}$ denotes the samples in the final stable stage and N_s is the number of samples in that stage.
- **Std** $|\alpha|$: the standard deviation of the absolute pendulum-angle error during the final stable stage,

$$\text{Std}\,|\alpha| = \sqrt{\frac{1}{N_s}\sum_{k\in\mathcal{S}}\left(|\alpha_k| - \text{Mean}\,|\alpha|\right)^2}.$$

- **Mean** $|PWM|$: the average absolute PWM control input during the final stable stage,

$$\text{Mean}\,|PWM| = \frac{1}{N_s}\sum_{k\in\mathcal{S}}|PWM_k|.$$

- **Std** $|PWM|$: the standard deviation of the absolute PWM input during the final stable stage. It reflects the fluctuation of the control effort after stabilization.
- **Maximum overshoot**: let k_s denote the final entry into the selected stabilization region. This entry is accepted only when the pendulum remains inside that region until the end of the run. The maximum overshoot is the largest absolute pendulum-angle deviation after this final entry,

$$\alpha_{\text{OS,max}} = \max_{k\geq k_s}|\alpha_k|.$$

  If the pendulum leaves the stabilization region after an earlier entry, that earlier entry is not used; a later final entry must be identified.

The same final stable-stage interval must be used when calculating Mean $|\alpha|$, Std $|\alpha|$, Mean $|PWM|$, Std $|PWM|$, and the maximum overshoot. The term "maximum overshoot" is retained here to denote the largest $|\alpha|$ observed after the final accepted entry into the stabilization region.

2.6 Quantitative continuous-versus-discrete comparison

For each controller, calculate the mean and standard deviation of every metric across the successful runs among the ten fixed-parameter trials. Also state the number of successful and failed runs. Report each numerical result in the form

$$\text{mean} \pm \text{standard deviation.}$$

Table 7: Continuous-design and discrete-design LQR simulation comparison.

Metric	Continuous-design K_c	Discrete-design K_d
Stable time/s	–	–
Mean $\lvert\alpha\rvert$/rad	–	–
Std $\lvert\alpha\rvert$/rad	–	–
Mean $\lvert PWM\rvert$	–	–
Std $\lvert PWM\rvert$	–	–
Max overshoot/rad	–	–

Discuss which controller performs better in terms of stabilization speed, pendulum-angle accuracy, response repeatability, control effort, PWM fluctuation and maximum overshoot.

Explain whether the observed difference is practically significant and whether it is consistent with the fact that K_c is designed from the continuous model, whereas K_d is designed directly from the ZOH-discretised model.

3 Part III: Physical-System LQR Results

The physical rotary inverted pendulum must be controlled using the discrete-design gain K_d. Record the exact gain vector sent to the STM32 and all relevant experimental settings.

Table 8: Physical-system LQR controller settings.

Setting	Value	Setting	Value
K_d		Control frequency	200 Hz
PWM limit		Swing PWM	
Enter angle		Exit angle	
Soft blend λ		Velocity LPF	
Potentiometer upright value		Downward value	
Motor sign		Experiment duration	

Complete ten independent physical-system experiments using the same fixed controller settings. Do not change K_d or any other controller parameter between runs. Record failed runs as exttFail and use "–" for unavailable stable-stage quantities.

Save every physical experiment as one matched CSV/PNG pair. A recommended naming convention is

```
lqr_hardware_discrete_01.csv
lqr_hardware_discrete_01.png
```

and similarly for runs 02–10.

The evaluation quantities and stable-stage definition must be identical to those used in the simulation section. In particular, the swing-up stage and soft-blend transition stage must not be included in the stable-stage statistics.

Table 9: Ten repeated physical-system experiments using the discrete-design gain K_d.

Trial	Stable time/s	Mean $\lvert\alpha\rvert$/rad	Std $\lvert\alpha\rvert$/rad	Mean $\lvert PWM\rvert$	Std $\lvert PWM\rvert$	Max overshoot/rad
1	–	–	–	–	–	–
2	–	–	–	–	–	–
3	–	–	–	–	–	–
4	–	–	–	–	–	–
5	–	–	–	–	–	–
6	–	–	–	–	–	–
7	–	–	–	–	–	–
8	–	–	–	–	–	–
9	–	–	–	–	–	–
10	–	–	–	–	–	–
Across-run mean	–	–	–	–	–	–
Across-run std.	–	–	–	–	–	–

Describe and analyse the following physical-system behaviour:

- repeatability across the ten fixed-parameter experiments;
- swing-up and capture behaviour;
- stabilization speed and stable pendulum-angle error;
- rotary-arm drift or excessive arm motion;
- oscillation and sensor-noise effects;
- PWM effort and actuator saturation;
- any unsuccessful capture attempts;
- any manual stop or other safety intervention.

Retain every unsuccessful run among the ten trials, report the total numbers of successful and failed runs, and explain the likely causes. If any controller parameter is changed, discard the mixed-parameter set and repeat all ten trials using one final fixed parameter set.

4 Part IV: Sim-to-Real Analysis

Compare the discrete-design nonlinear simulation with the physical experiment. Use matched K_d, PWM limit, switching thresholds, blend and duration wherever possible.

Table 10: Matched discrete-LQR simulation and physical-system comparison

Metric	Simulation	Physical system	Relative difference / %
Stable time / s			
Mean $\|\alpha\|$ / rad			
Std. $\|\alpha\|$ / rad			
Mean |PWM|			
Std. |PWM|			
Max overshoot / rad			

Explain the discrepancies using evidence from the logs. Consider model-parameter error, unmodelled friction, backlash, PWM dead zone, motor saturation, potentiometer calibration, encoder resolution, numerical velocity estimation, communication/logging effects and trial-to-trial variation. Do not merely state that "the real system has noise."

5 Part V: LQR versus the Group's Class-8 Dual PID

Use the group's own best Class-8 dual-PID parameters. State them explicitly, including both integral gains. If $K_i = 0$, convert the PID stabiliser into its equivalent full-state feedback vector

$$K_{\mathrm{PID,eq}} = [-K_{p,\theta}, -K_{d,\theta}, -K_{p,\alpha}, -K_{d,\alpha}].$$

Explain that this equivalence concerns the local stabilising law; LQR differs in how the gain is calculated.

5.1 Simulation comparison

Use identical nonlinear-plant, initial-condition and noise settings for LQR and PID.

Report each quantitative result in the form

mean ± standard deviation.

Table 11: LQR and Class-8 dual-PID simulation comparison

Metric	Discrete LQR	Group's dual PID
Stable time / s		
Mean stable-stage $\lvert\alpha\rvert$ / rad		
Std. stable-stage $\lvert\alpha\rvert$ / rad		
Max overshoot / rad		
Mean stable-stage |PWM|		
Std. stable-stage |PWM|		

Let k_s be the final entry into the specified capture region $|\alpha| \le \alpha_{\text{cap}}$. This entry is valid only if the pendulum remains inside the region until the end of the run. If it leaves after an earlier entry, that earlier entry is rejected and a later final entry is used. The reported maximum overshoot is

$$\alpha_{\text{OS,max}} = \max_{k \ge k_s} |\alpha_k|.$$

5.2 Physical-system comparison

Compare the new discrete-LQR physical-system results with the group's own Class-8 dual-PID physical-system results.

The comparison must use the same evaluation quantities, final-entry rule and stable-stage definition adopted in the previous sections. In particular, the stable stage begins at the final entry into the specified stabilization region, after which the pendulum remains inside that region until the end of the run. Swing-up and soft-blend data must not be included in the stable-stage statistics.

Use ten fixed-parameter physical-system runs for each controller and retain any failed runs in the record. Calculate the quantitative statistics from the successful runs and report each result in the form

$$\text{mean} \pm \text{standard deviation}.$$

State any unavoidable differences in calibration, sensor condition, initial condition, hardware condition, or other experimental settings.

Table 12: LQR and Class-8 dual-PID physical-system comparison

Metric	Discrete LQR	Group's dual PID
Stable time / s	–	–
Mean stable-stage $\lvert\alpha\rvert$ / rad	–	–
Std. stable-stage $\lvert\alpha\rvert$ / rad	–	–
Max overshoot / rad	–	–
Mean stable-stage |PWM|	–	–
Std. stable-stage |PWM|	–	–

Discuss the comparison from both performance and engineering perspectives.

Explain whether the observed differences are practically significant. If the two experiments were conducted under different calibration or hardware conditions, discuss how these differences may affect the fairness of the comparison.

6 Part VI: Summary

Summarise the main findings without repeating the entire report.